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Influence of the linear mode coupling on the nonlinear impairments in few-mode fibers

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Abstract

This paper is focused on the influence of the linear mode coupling caused by the fiber bending on the nonlinear distortions in a mode-division multiplexed system. The system under test utilizes the fundamental Gaussian mode and the conjugated first-order vortex modes propagating in the step-index fiber at the same wavelength. For such kind of system, the nonlinear impairments are caused mainly by the cross-phase and self-phase modulations. Propagation of the modal composition is described by the system of generalized coupled nonlinear Schrödinger equations, which serves as a basis of our simulations. Considering the nonlinear operator analytically, we show that it reaches its maximum value due to the power transfer between mode channels caused by the linear mode coupling. Simulation results for equal initial powers in NRZ-coded mode channels demonstrate that nonlinear signal impairments increase significantly for all mode channels in the case of strong linear mode coupling. In the case of weak linear coupling, the increase of nonlinear impairments was also observed, but this effect was appreciably weaker. Moreover, simulations show that the effect described above is stronger for the first-order modes than for the fundamental mode.

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Keywords: vortex modes; few-mode fibers; Kerr-nonlinearities; mode division multiplexing

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1. Introduction

Recent publications in the field of the optical communications capacity show that technologies of single-mode fiber optics are not able to satisfy the growing traffic demand [1, 2]. Optical spatial division multiplexing (SDM) is the promising technology to avoid the capacity crunch, therefore it attracts great attention in recent years [3, 4]. SDM systems are being considered for long-haul systems using coherent detection or short-range systems using direct detection. This technology allows increasing the transmission capacity by multiplexing several signals in the modes of the fiber, so it is often called mode-division multiplexing (MDM). One of the main principal problems for SDM systems is the linear mode coupling that defines strong cross-talk between mode channels [5]. Mode coupling is caused by fiber bending, twisting and inherent optical fiber fabrication defects [6]. It is well known that in the realistic conditions there is strong mode coupling between modes having similar wavenumbers, but their coupling to the modes whose wavenumber is significantly different is much weaker. Previously it was shown that linear mode coupling can be compensated by means of digital signal processing using the “multiple input – multiple output” (MIMO) technology [7, 8]. Alternative approach for the mode coupling compensation can be based on the spatial mode decomposition by means of computer-generated hologram [9]. Another fundamental problem of SDM systems is the nonlinear coupling between the propagating modes [10, 11]. It is evident that nonlinear distortions in the mode channels may sufficiently decrease the efficiency of the linear mode coupling compensation. In this paper, we consider the interaction of the linear and nonlinear mode coupling effects in the case of simultaneous propagation of few spatial fiber modes. In the case of silica fibers, the nonlinear impairments are defined by the third-order Kerr nonlinearities. A number of recent publications were focused on the intermodal four-wave mixing between simultaneously propagating modes on the different wavelengths, that may provide for phase matching [12, 13]. For SDM systems design, the important topic is the investigation of nonlinear transmission of few modes at the same wavelength, when the cross-phase modulation will be the significant origin of nonlinear cross-talk.

Following the approach used to design the mode-division multiplexed system in previous experimental work [9], we investigated signal transmission in standard step-index fiber G.652 in the wavelength region near 850 nm. Such fiber provides propagation of three spatial linearly polarized modes (here we do not consider polarization effects): the two modes LP₁₁ and the fundamental mode LP₀₁.

It was previously shown [14] that measured strength of the nonlinear interaction between the fundamental mode and the first-order modes through cross-phase modulation corresponds to the analytical model based on the propagation of vortex modes LP₁₁⁺ and LP₁₁[−], but not the modes LP_{11a} and LP_{11b}. Taking this into account, we simulate degenerate LP₁₁ modes in the basis of vortex modes LP₁₁⁺ and LP₁₁[−]. Note that the basis of modes LP₁₁⁺ and LP₁₁[−] has the property of the same intensity pattern for both degenerate modes and these modes have a helical transverse phase structure of $\exp(il\phi)$, where ϕ is the transverse azimuthal angle and l is the azimuthal order, which is an unbounded integer that indicates the topological charge of the mode.

2. Theoretical background

In the case of simultaneous propagation of N spatial modes one can consider electromagnetic wave in frequency domain as a sum of N items

$$E(r, \phi, z, \omega) = \sum_{p=1}^N \exp(i\beta_p(\omega)z) A_p(z, \omega) F_p(r) \exp(im_p\phi), \quad (1)$$

where p is the mode number, β_p is the propagation constant of the p -th spatial mode, $F_p(r, \phi) = F_p(r)\exp(im_p\phi)$ is the transverse distribution of the p -th mode field, m_p is the integer number defining azimuthal order of the p -th mode and A_p is the slowly varying amplitude of the p -th mode.

Transmission of the mode superposition in silica fiber in time domain is described by the system of generalized coupled nonlinear Schrödinger equations that might be written as [11]

$$\frac{\partial A_p}{\partial z} = i(\beta_{0p} - \beta_r)A_p - \left(\beta_{1p} - \frac{1}{v_{gr}} \right) \frac{\partial A_p}{\partial t} - i \frac{\beta_{2p}}{2} \frac{\partial^2 A_p}{\partial t^2} - \frac{\alpha}{2} A_p + i \sum_{lmn} f_{lmnp} \gamma_l A_l^* A_m A_n + i \sum_m q_{mp} A_m, \quad (2)$$

where A_p is the slowly varying envelope expressed in a reference moving frame at a group velocity v_{gr} , β_r is the reference propagation constant and γ is the nonlinear parameter.

$$\beta_{0p} = \beta_p(\omega_0), \beta_{1p} = \partial \beta_p / \partial \omega|_{\omega_0}, \beta_{2p} = \partial^2 \beta_p / \partial \omega^2|_{\omega_0} \quad (3)$$

are respectively the propagation constant, inverse group velocity and group-velocity dispersion (GVD) of the p -th spatial mode.

Linear mode coupling in Eq. 2 is defined by the coefficients

$$q_{mp}(z) = \frac{k_0}{2n_{eff}(I_m I_p)^{1/2}} \iint \Delta n^2(x, y, z) F_m F_p dx dy, \quad (4)$$

where the intensity normalization terms are defined as

$$I_m = \bar{n}_m / \bar{n}_{eff} \iint F_m^2(x, y) dx dy. \quad (5)$$

Here n_{eff} and n_m are the effective refractive indices for the fundamental mode and for the m -th mode respectively.

Linear coupling terms appear as a result of the perturbation of the refractive index $\Delta n(x, y, z)$. In the case of an ideal fiber no linear coupling occurs between the spatial modes because $\Delta n^2(x, y, z) = 0$.

For mode coupling simulation, the fiber is conventionally approximated as a concatenation of sections with constant geometric parameters such as ellipticity, curvature radius, etc. It was previously reported that the main reason of linear coupling in optical fibers is the fiber bending [5] with the corresponding coupling coefficients [15]

$$q_{\mu\nu}^{(k)} = \frac{\gamma_\mu \gamma_\nu}{2n_1 k_0 a \sqrt{e_p e_m}} \cdot \frac{\xi^{(k)} (\delta_{p,m-1} + \delta_{p,m+1} + \delta_{p,1-m})}{(\beta_\mu - \beta_\nu)^2} \Upsilon_{\mu\nu}, \quad (6)$$

where n_1 is the refractive index of the fiber core in the point $z = 0$, $\xi(k)$ is the curvature of the k -th section, p and m are the mode azimuthal indices, δ is the Kronecker delta function, $e_m = 2$ for $m = 0$ and $e_m = 1$ in other cases,

$$\Upsilon_{\mu\nu} = \frac{J_p(\kappa_\mu a) J_m(\kappa_\nu a)}{\sqrt{|J_{p-1}(\kappa_\mu a) J_{p+1}(\kappa_\mu a) J_{m-1}(\kappa_\nu a) J_{m+1}(\kappa_\nu a)|}}. \quad (7)$$

In Eq. 7 $J_l(\kappa_\mu a)$ is the Bessel function with κ_μ defined as

$$\kappa_\mu = \sqrt{(k_0 n_1)^2 - \beta_\mu^2}. \quad (8)$$

Nonlinear mode coupling in Eq. 2 is defined by coefficients

$$f_{lmnp} = \frac{A_{eff}}{(I_l I_m I_n I_p)^{1/2}} \iint F_l^* F_m F_n F_p^* dx dy, \quad (9)$$

where A_{eff} is the effective mode area [16].

Taking into account the expression given in Eq. 1 for the overlap integral defined by Eq. 9, one can define condition of the non-zero value [17]:

$$f_{lmnp} = 0 \quad \text{for} \quad -m_l + m_m + m_n - m_p \neq 0. \quad (10)$$

Using this condition we can exclude coefficients that have non-zero values because of numerical errors. As a result, we have a set of coefficients, presented in the Tab. 1.

Table 1. Non-zero nonlinear coefficients for a few-mode fiber with 3 spatial modes.

LP mode indices	f_{lmnp}
LP01,LP01,LP01,LP01	1..000
LP11-,LP11-,LP11-,LP11-	0.6975
LP11+,LP11-,LP11+,LP11-	
LP11-,LP11+,LP11-,LP11+	
LP11-,LP11-,LP11+,LP11+	
LP11+,LP11+,LP11+,LP11+	
LP01,LP01,LP11-,LP11-	0.6595
LP01,LP11-,LP01,LP11-	
LP11+,LP01,LP01,LP11-	
LP11-,LP01,LP11-,LP01	
LP01,LP11+,LP11-,LP01	
LP11-,LP11-,LP01,LP01	
LP11+,LP11+,LP01,LP01	
LP01,LP11-,LP11+,LP01	
LP11+,LP01,LP11+,LP01	
LP11-,LP01,LP01,LP11+	
LP01,LP11+,LP01,LP11+	
LP01,LP01,LP11+,LP11+	

Let us consider in more detail the influence of two mode channels on the third channel through the Kerr-nonlinearities. For example, for the LP11+ mode channel, the nonlinear operator in Eq. 2 will be defined as

$$\hat{N} = f_{1223} A_1^* A_2^2 + (f_{1313} + f_{1133}) |A_1|^2 A_3 + (f_{2323} + f_{2233}) |A_2|^2 A_3 + f_{3333} |A_3|^2 A_3, \quad (11)$$

where the indices 1, 2, 3 correspond to the modes LP11-, LP01, LP11+ respectively.

The first term in Eq. 11 corresponds to the intermodal four-wave mixing, but because of the phase mismatch this term is rapidly oscillating with low amplitude.

The fourth term corresponds to the self-phase modulation. Its effectiveness for the first-order modes is lower than for the fundamental mode because of the corresponding values given in the Tab. 1.

The second and the third terms describe cross-phase modulation. Note that the cross-phase modulation efficiency is defined not only by the values of nonlinear coupling coefficients, but also by the difference between the group velocities of the interacting modes [14]. For the fiber under test the group velocities of the fundamental mode and the first-order modes are considerably different, therefore the interaction between the fundamental mode and the vortex modes through the cross-phase modulation is weaker than that between the conjugated modes. So the main

sources of nonlinear impairments for the signal with envelope $A_3(z)$ in Equation 15 are the second and the fourth terms.

As noted earlier, there is a strong linear coupling between the conjugated modes. To estimate the influence of the linear coupling on the nonlinear impairments, let us consider a simplified model. Assume that initially the optical power in each of the mode channels LP11+ and LP11– is P_0 . Linear coupling between these modes is stronger than that between each of them and the fundamental mode, so over some propagation distance their common power may be considered to be a constant equal to $2P_0$. Denoting

$$f_{1313} = f_{1133} = f_{3333} = f, \quad (12)$$

one can derive amplitude of the nonlinear operator in the form

$$|\hat{N}| = 4fP_0 |A_3| - f|A_3|^3. \quad (13)$$

It is easy to see that the function given in Eq. 13 reaches its maximum value when $|A_3| = \sqrt{(4P_0/3)}$. The physical meaning of this result is clear: the most significant distortions caused by Kerr-nonlinearities appear when the channel power increases due to the linear mode coupling. Thus, for initially close power values in the conjunct mode channels, linear coupling causes increasing of the nonlinear impairments.

3. Simulation results

We solved Equations 2 numerically using split-step Fourier method. Linear mode coupling is considered by multiplying the field vector $\mathbf{A}(t)$ at each step by the matrix

$$\mathbf{T}_\Delta = \exp[i(\mathbf{B}_0 + \mathbf{Q}(z))\Delta z], \quad (14)$$

where \mathbf{B}_0 is the square matrix with main diagonal elements defined by the first term of the right side of Equation 2, $\mathbf{Q}(z)$ is the square matrix with elements defined from Equation 6 and Δz is the simulation step.

For estimation of the nonlinear impairments, Equations 2 were solved twice for each set of random parameters defining sections of simulated fiber: taking into account nonlinear terms of equations, and leaving them out of account. Thus, two envelopes of the field were found using this method. First of them takes into account both dispersion and nonlinear equation terms, while the second one takes into account only dispersion terms. Then RMS variance of these two envelopes was calculated.

In the first experiment series, all mode channels were amplitude-modulated using NRZ-signal consisted of alternating zeros and ones with pulse duration set to 100 ps. Note that such kind of modulation pattern is not realistic but provides the worst case of nonlinear distortions caused by cross-phase modulation because phase distortions correspond to pulse edges. The power of all three signals was the same. Simulated fiber length was set to 2500 m. Linear mode coupling was simulated by splitting the fiber on 5-cm-length sections, that corresponds to the case of strong mode coupling [5]. Simulation results are given on Fig. 1.

In the second experiment series linear mode coupling was simulated by splitting the fiber on 5-m-length sections, that corresponds to the case of weak mode coupling. Simulation results are given on Fig. 2.

Presented results show that linear mode coupling lead to increasing nonlinear impairments in all mode channels in the both cases of strong and weak mode coupling. However, weak linear coupling affects the nonlinearities much less. Moreover, simulation results show that the nonlinear impairments in the fundamental mode channel are less affected by the linear coupling than those in the first-order modes. It can be explained by the difference between the group velocities that decrease distortions due to the cross-phase modulation, that was mentioned above.

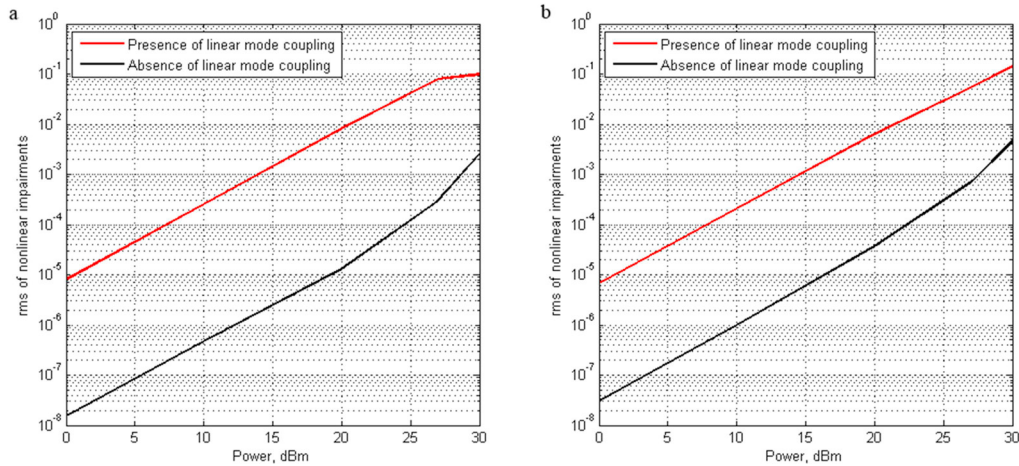


Fig. 1. Nonlinear distortions in mode channels in strong-coupling regime: (a) LP11; (b) LP01.

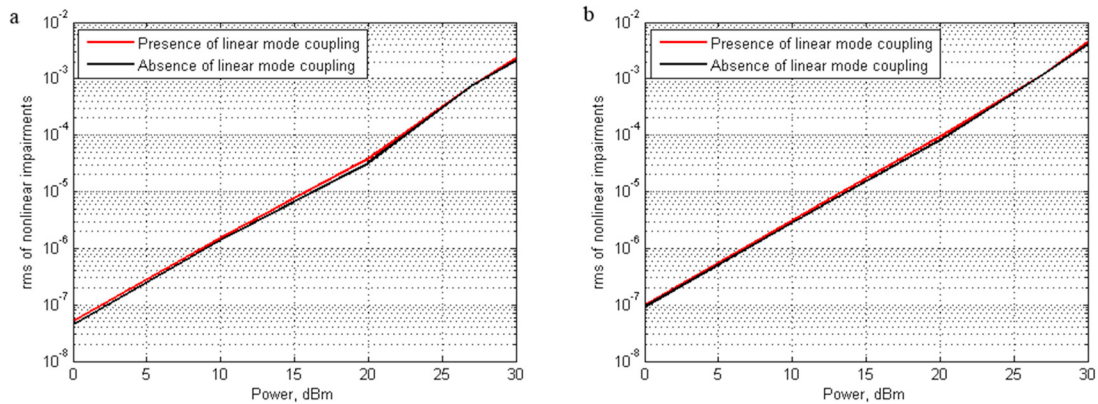


Fig. 2. Nonlinear distortions in mode channels in weak-coupling regime: (a) LP11; (b) LP01.

4. Conclusions

In this paper we have considered SDM system using the standard step-index fiber that provides transmission of three modes in the 850 nm wavelength region. Interaction between the fundamental mode LP01 and the vortex modes LP11+ and LP11– have been considered. We have simulated propagation of NRZ bit streams with initially equal powers in each mode channel in the presence of fiber dispersion, nonlinearity and linear mode coupling. We have shown analytically that the power transfer between the mode channels may lead to the increasing nonlinear distortions. Simulation results for 2.5-km fiber link with strong mode coupling confirm that the presence of linear mode coupling considerably increases the nonlinear signal impairments.

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